Assignment 2

Coverage: 15.2 and 15.3 in Text.

Exercises: 15.2. no 20, 22, 27, 35, 39, 44, 48, 55, 61, 65, 79. 15.3. no 3, 6, 10, 11, 13, 15, 17, 21, 23, 26, 30.

Submit 15.2 no 27, 48, 65, 15.3 no 15, 21 by January 31, 2023.

Supplementary Problems

1. Let S be the sector bounded by the straight line $y = (\tan\alpha)x$, the positive x-axis and the circle $x^2 + y^2 = r^2$. Show that its area is given by $\alpha r^2/2$. To simply the calculation, you may assume $\alpha \in (0, \pi/2]$.

2. Let f and g be continuous on the region D. Deduce the inequality

$$2 \iint_D |fg|\, dA \le \alpha^2 \iint_D f^2\, dA + \frac{1}{\alpha^2} \iint_D g^2\, dA \ ,$$

where α is a positive number. Hint: Use $(\alpha f(x) - \alpha^{-1} g(x))^2 \ge 0$.

3. Setting as in (2), prove the Cauchy-Schwarz inequality:

$$\iint_D |fg|\, dA \le \left(\iint_D f^2\, dA \right)^{1/2} \left(\iint_D g^2\, dA \right)^{1/2} \ .$$

Hint: Choose a suitable α in (2).

4. Let f be a non-negative continuous function on D and p a positive number. Show that

$$m \le \left(\frac{1}{|D|} \iint_D f^p\, dA \right)^{1/p} \le M \ ,$$

where m and M are respectively the minimum and maximum of f and $|D|$ is the area of D.